# Why?

Digital logic allows us to build large but reliable computer systems. In various stages of the design process, different representations of the same logic function may be most useful. You’ll convert between those different representations in this activity.

# What you need to already know

Before doing this activity, you should be able to calculate the output of the common logical operators with 1 or 2 inputs, specifically AND, OR, NOT, and XOR.

# Model 1: The truth tables of logical operators

By exploring this Model, you’ll learn to...

* Describe how truth tables are related to logical operators

Legend:

* unshaded column: input
* shaded column: output
* 1 = True
* 0 = False

The tables below represent Fred, Geri, and Harriet’s preferences for pizza toppings.

C = the pizza has cheese

M = the pizza has mushrooms

output = that person is satisfied

Fred

|  |  |  |
| --- | --- | --- |
| **C** | **M** | **f(C,M)** |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

Geri

|  |  |  |
| --- | --- | --- |
| **C** | **M** | **g(C,M)** |
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

Harriet

|  |  |
| --- | --- |
| **C** | **h(C)** |
| 0 | 1 |
| 1 | 0 |

1. What do shaded columns represent? What do unshaded columns represent?
2. What is the output of Fred when...
   1. The pizza doesn’t have cheese, doesn’t have mushrooms?
      1. 0
   2. The pizza has cheese, has mushrooms?
      1. 1
   3. The pizza has cheese, doesn’t have mushrooms?
      1. 1
   4. The pizza doesn’t have cheese, has mushrooms?
      1. 1
3. Summarize Fred’s preference for pizza.

He likes all pizza that has at least one topping.

1. Which familiar logical operator is **f(C,M)**?

AND OR NOT XOR

1. What is the output of Geri when...
   1. The pizza doesn’t have cheese, doesn’t have mushrooms?
   2. The pizza has cheese, has mushrooms?
   3. The pizza has cheese, doesn’t have mushrooms?
   4. The pizza doesn’t have cheese, has mushrooms?
2. Summarize Geri’s preference for pizza.
3. Which familiar logical operator is **g(C,M)**?

AND OR NOT XOR

1. What is the output of Harriet when...
   1. The pizza doesn’t have cheese?
   2. The pizza has cheese?
2. Summarize Harriet’s preference for pizza.
3. Which familiar logical operator is **h(C)**?

AND OR NOT XOR

1. How do the tables in Model 1 relate to the logical operators you picked for each person?

# Read This!

Each of the tables in Model 1 is called a **truth table**, which is related to a logical operator in the way you described above. Truth tables can represent not just familiar logical operators (AND, OR, NOT, and XOR) but also *any* Boolean function!

# Model 2: The rows of a truth table

By exploring this Model, you’ll learn to...

* Determine the number of rows in a truth table

Legend:

* unshaded column: input
* shaded column: output

(f)

|  |  |  |
| --- | --- | --- |
| **A** | **B** | **f(A,B)** |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

(g)

|  |  |  |
| --- | --- | --- |
| **A** | **B** | **g(A,B)** |
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

(h)

|  |  |
| --- | --- |
| **A** | **h(A)** |
| 0 | 1 |
| 1 | 0 |

(k)

|  |  |  |  |
| --- | --- | --- | --- |
| **A** | **B** | **C** | **k(A,B,C)** |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

1. How many inputs, outputs, and rows does each truth table have?

|  |  |  |  |
| --- | --- | --- | --- |
| **Truth Table** | **number of input columns** | **number of output columns** | **number of rows** |
| f | 2 | 1 | 4 |
| g | 2 | 1 | 4 |
| h | 1 | 1 | 1 |
| k | 3 | 1 | 8 |

1. For a truth table, how is the number of rows related to the number of input columns and number of output columns?

Rows = 2^(number of input columns)

**Share!** Write your team’s answer to #13 on the board.

1. The truth tables in Model 2 all use a convention for the order of the rows. What is that order? Why might this ordering convention helpful?

# Exercises

1. Here is *only* the heading (no rows shown) for some other truth table. Calculate how many rows this truth table should have.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **A** | **B** | **C** | **D** | **E** |

# Extension Questions

1. How many *unique* truth tables are there that have 2 input columns and 1 output column? How many for N input columns? Show your work using diagrams and/or calculations.

2^2^N

# Model 3: Converting Boolean equations to truth tables

By exploring this Model, you’ll learn to...

* Translate a Boolean equation to a truth table

legend:

* unshaded column: input
* shaded column: output
* + OR
* AND

|  |  |  |
| --- | --- | --- |
| **A** | **B** | **f(A,B)** |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

|  |  |  |
| --- | --- | --- |
| **A** | **B** | **g(A,B)** |
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

|  |  |
| --- | --- |
| **A** | **h(A)** |
| 0 | 1 |
| 1 | 0 |

1. What is the mathematical notation for AND, OR, and NOT?

AND = , OR = +, NOT =

1. What Boolean operator (AND, OR, NOT, XOR) does each *equation* in Model 3 represent?

f. OR

g. AND

h. NOT

1. **Check Yourself!** Does your answer to #18 match your answers to #4,7,10?

sure

1. Using Model 3, determine:
2. Describe the relationship between an equation and its truth table. Make sure to mention the rows and columns of the truth table.

The truth table for an equation gives all possible outcomes for all possible inputs. Each row is one set of input/output, each column represents all …

1. What is the domain of the variables in the equations in Model 3?

Domain: 0 to 1

1. In each of the 3 equations in Model 3, what is the codomain (or equivalently, in this case, *range*) of each side of the equation?
2. left hand side, i.e. ? right hand side, i.e. ?

0-1 0-1

1. left hand side? right hand side?

0-1 0-1

1. left hand side? right hand side?

0-1 0-1

1. Using your answers to #22, 23, give the important qualities of equations like those in Model 3.

Only 1’s and 0’s are used.

# Read This!

The equations in Model 3 are examples of **Boolean equations**. A Boolean equation equates (using “=” ) two **Boolean expressions**.

# Exercises

1. Finish the truth table for this Boolean equation. Refer to Model 3 if you forget what the notation means.

|  |  |  |  |
| --- | --- | --- | --- |
| **A** | **B** | **C** | **w(A,B,C)** |
| 0 | 0 | 0 |  |
| 0 | 0 | 1 |  |
| 0 | 1 | 0 |  |
| 0 | 1 | 1 |  |
| 1 | 0 | 0 |  |
| 1 | 0 | 1 |  |
| 1 | 1 | 0 |  |
| 1 | 1 | 1 |  |

1. Draw and fill in the truth table for this equation.

# Extension Questions

1. Does a truth table correspond to exactly one Boolean equation? If yes, give justification in complete sentences. If no, then give a counterexample.

No, because you can simplify equations to get different equations.

# Model 4: Translating truth tables to Boolean equations

By exploring this Model, you’ll learn to...

* Translate a truth table to a Boolean equation

|  |  |  |
| --- | --- | --- |
| **A** | **B** | **f(A,B)** |
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

Algorithm: (shortcut of #28-#31).

1. Find the rows with a 1 in the output
2. For each of those rows, write a product
   1. Not(A)\*Not(B)
   2. Not(A)\*B
   3. A\*B
3. Or all the terms together.
   1. Not(A)\*Not(B) + Not(A)\*B + A\*B = f(A,B)

**Manager:** assign #28-30 to individual team members.

1. Look at the *first row* of the truth table. Write a Boolean expression in terms of A, B such that the expression *only* evaluates to 1 when A and B have the values in that row. You may only use and   operators.
2. Look at the *second row* of the truth table. Write a Boolean expression in terms of A, B such that the expression *only* evaluates to 1 when A and B have the values in that row. You may only use and   operators.
3. Look at the *fourth row* of the truth table. Write a Boolean expression in terms of A, B such that the expression *only* evaluates to 1 when A and B have the values in that row. You may only use and   operators.
4. Share your answers to #28-30 and check for errors.
5. Write a Boolean equation for that includes your expressions in #28-30. You may use any Boolean operators.
6. **Check yourself!** Does your Boolean equation in #32 match the truth table? Once it does, copy your equation into the blank on Model 4.
7. The procedure you followed in #28-32 generalizes to any truth table! Explain this procedure for translating a truth table into a Boolean equation.

**Share!** Send a representative to another team to compare your answers to #32, 34. Report back to your team.

# Read this!

The procedure you came up with to translate a truth table to a Boolean equation is called ***sum of products*.**

1. Considering the particular notation we are using for Boolean expressions, why is the procedure called *sum of products*?

# Exercises

1. Using sum of products, translate the following truth table to a Boolean equation.

|  |  |  |  |
| --- | --- | --- | --- |
| **A** | **B** | **C** | **D** |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |

1. Why does the sum of products ignore the rows where output = 0?
2. Explain why sum of products works. *HINT*: why did #28-30 ask you to write an expression that *only* evaluates to 1 when A and B have the values in that row?

**Discuss!** Make certain that everyone in your team agrees on and understands #37, 38. These are difficult questions.

**Share!** Send a representative to another team to compare your answers to #36-38. Report back to your team.

# Extension Questions

1. There is also a procedure called ***product of sums***. Translate the truth tables from Model 4 and #36 using product of sums. Try to figure out the procedure for product of sums yourself first. If you get stuck you can look it up.
2. You can always use either sum of products or product of sums.When would you prefer to use one or the other?
3. In #38 you explained why sum of products works. Why does *product of sums* work?